Managing Space for Finite-State Verification

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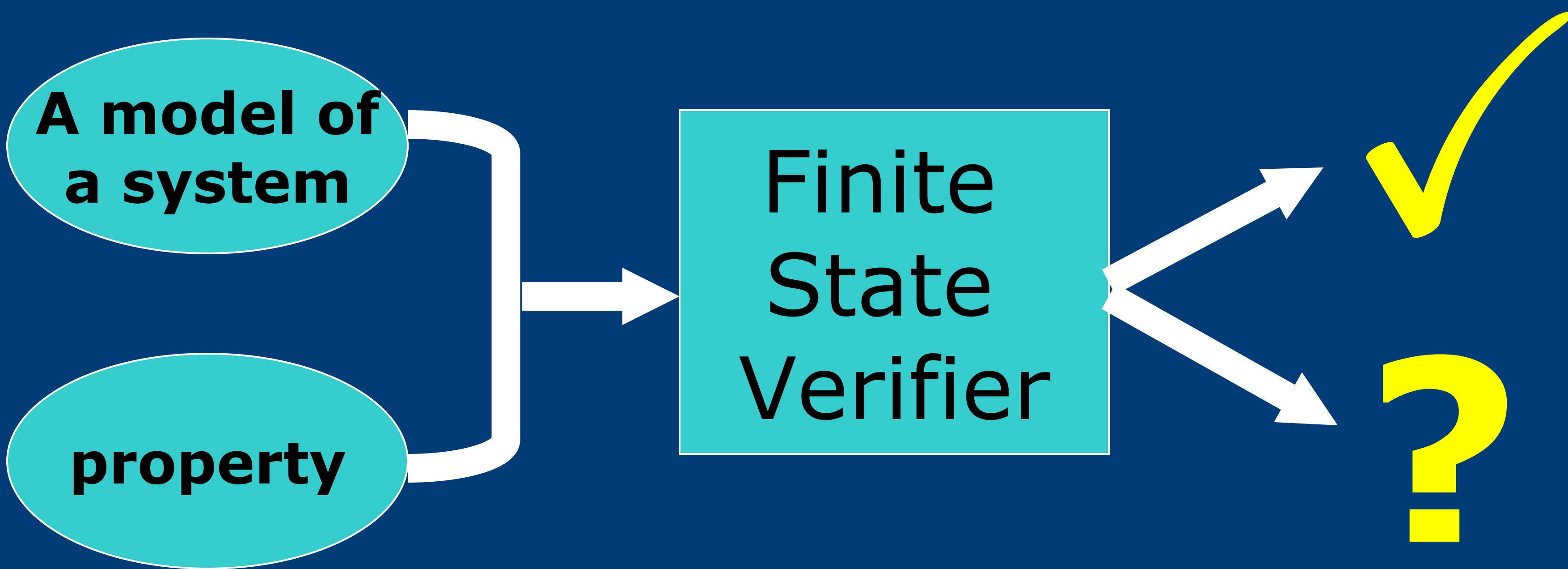
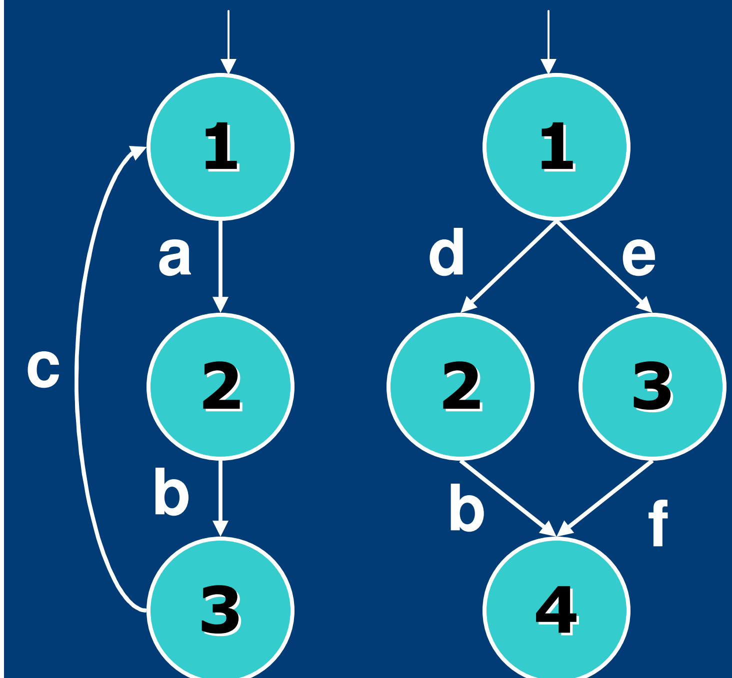
Finite-State Verification (FSV)

- Attempts to prove properties about a model of a system



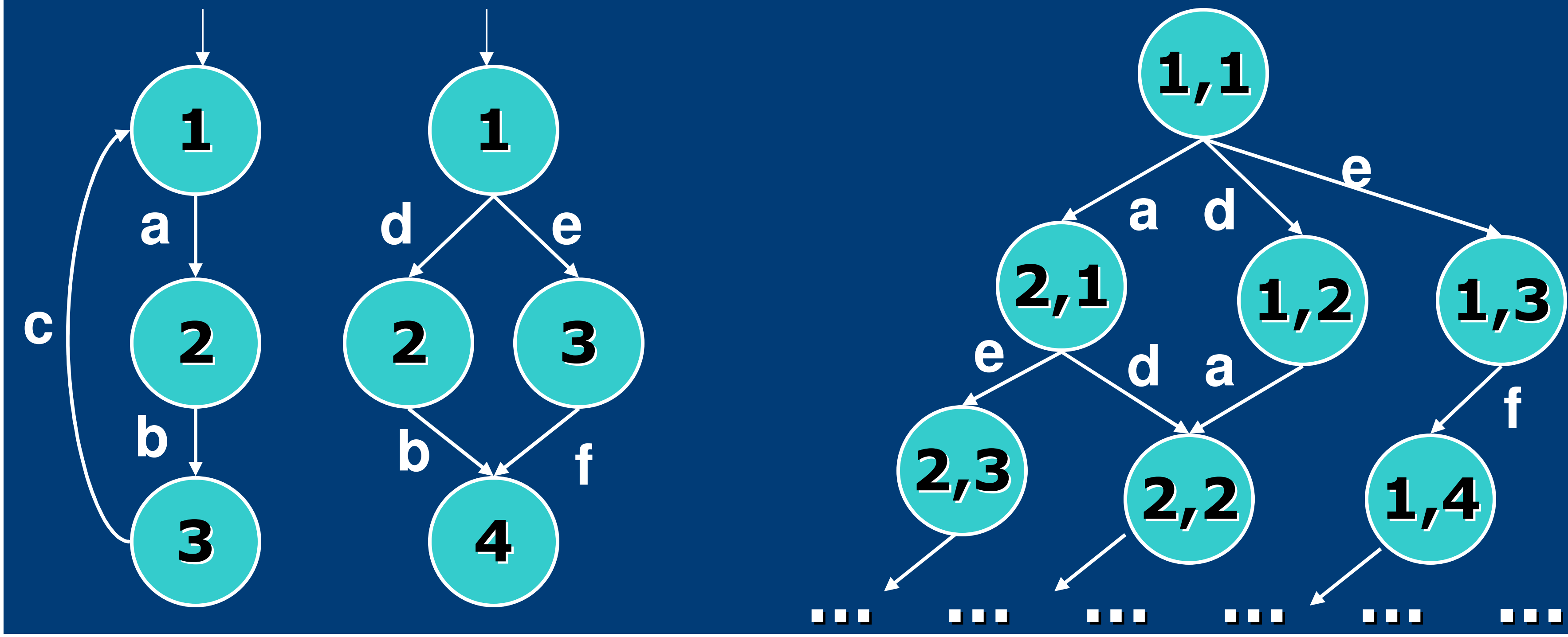
Models in FSV

- Typically, a system is modeled by a set of processes where each process is modeled by a Finite State Automaton (FSA)



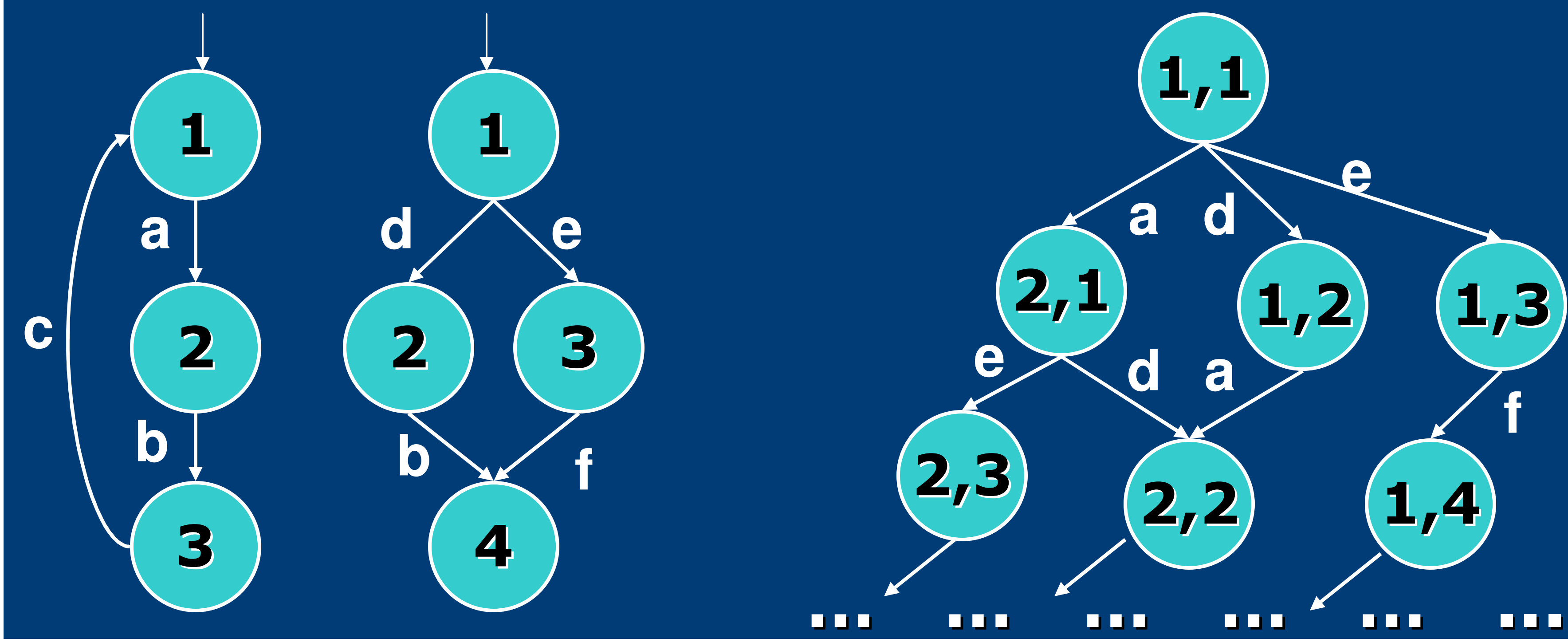
Models in FSV

- The overall system behavior is represented by the cross product FSA of all the process FSAs



State Explosion Problem

- But the number of *reachable states* in the cross product FSA could be exponentially large w.r.t. the number of process FSA states



Symbolic Approaches

- Use symbolic data structures to compactly represent subsets of reachable states
  - Shown to be useful for reducing the impact of the State Explosion problem in hardware verification

- But their value for software verification is less clear

Our Work

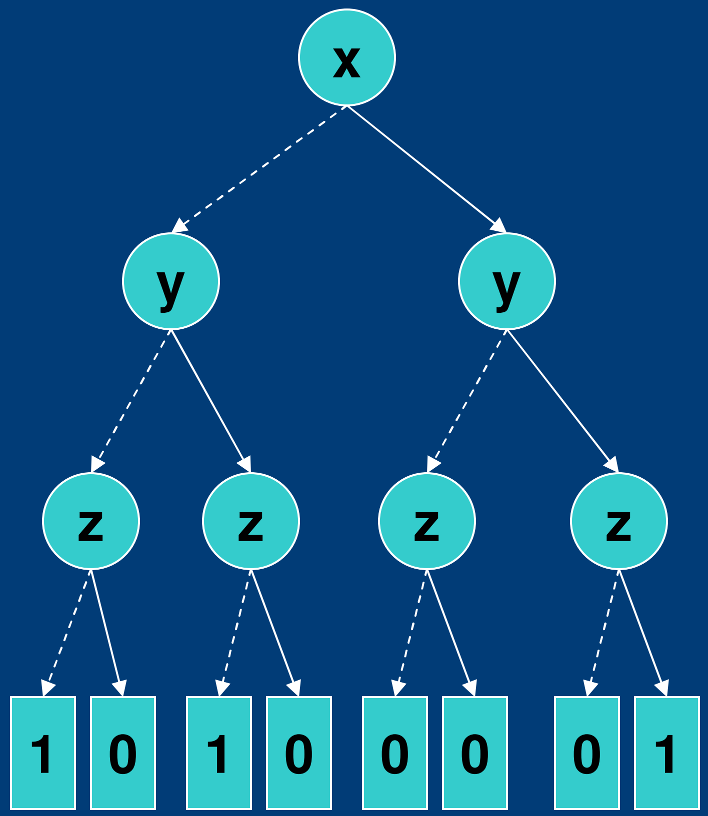
- Explored using two symbolic data structures
  - Binary Decision Diagrams (BDDs)
  - Zero-suppressed BDDs (ZDDs)

  in two finite state verifiers
  - LTSA
  - FLAVERS
- Developed a modified algorithm for computing the reachable states for ZDDs
- Developed a heuristic for variable ordering for BDDs/ZDDs
- Found an approach that usually ***increases*** the size of the systems that can be verified and ***decreases*** the time of the analysis

Outline

- Overview of BDDs and ZDDs
- Applying BDDs and ZDDs in FSV
- Experimental methodology and results
- Conclusions

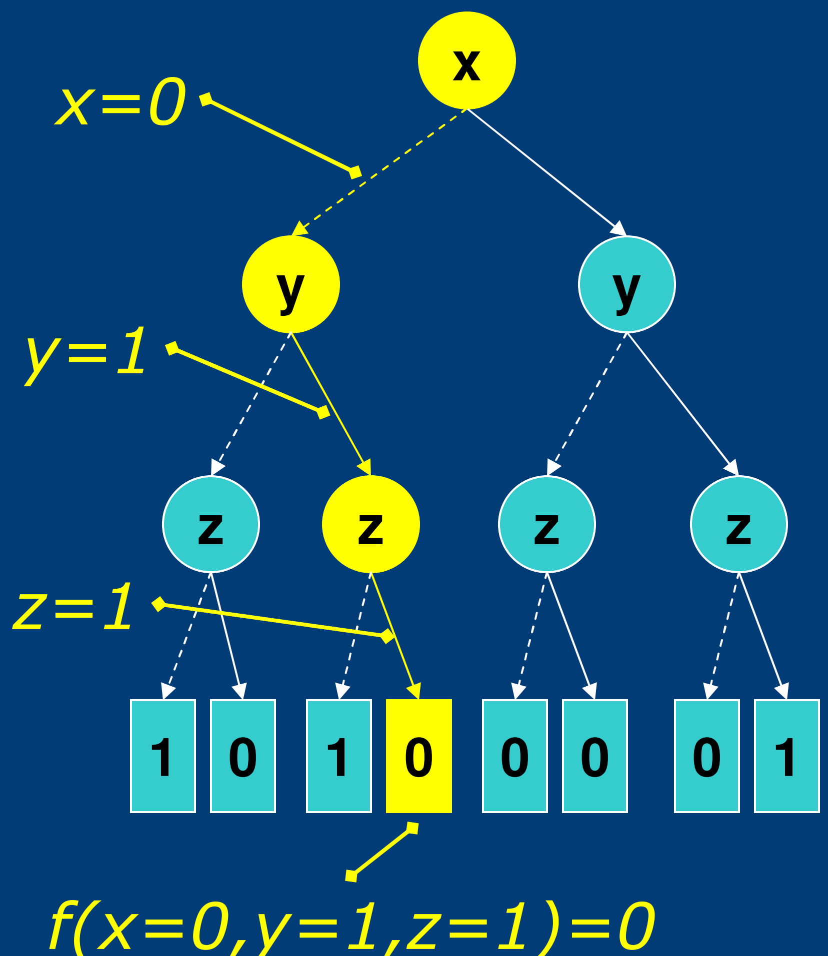
Binary Decision Trees



- From which BDDs and ZDDs are derived
- Represent Boolean functions

Binary Decision Trees

$f(x,y,z) = (\neg x \wedge \neg z) \vee (x \wedge y \wedge z)$



- Each path represents an assignment
  - 0-edge: the variable is assigned to 0
  - 1-edge: the variable is assigned to 1
  - The label of the terminal vertex determines the function value

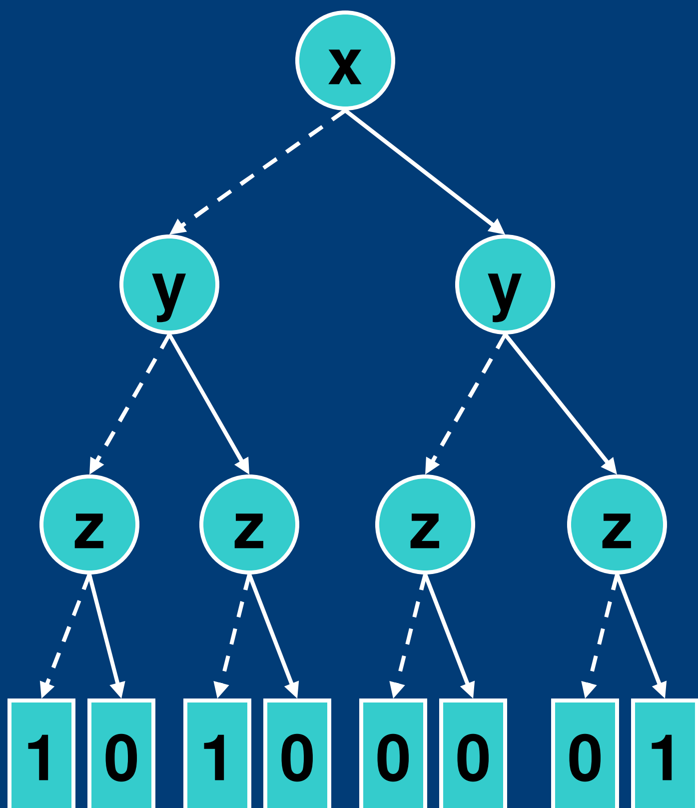
Exponentially Large!

Deriving BDDs and ZDDs

- BDDs [Bryant] and ZDDs [Minato] are reduced Binary Decision Trees
  - Both diagrams are derived by applying 3 reduction rules
  - The first 2 rules are the same for both diagrams
  - Only the third rule is different

Deriving BDDs and ZDDs

- Rule 1 merges terminal vertexes



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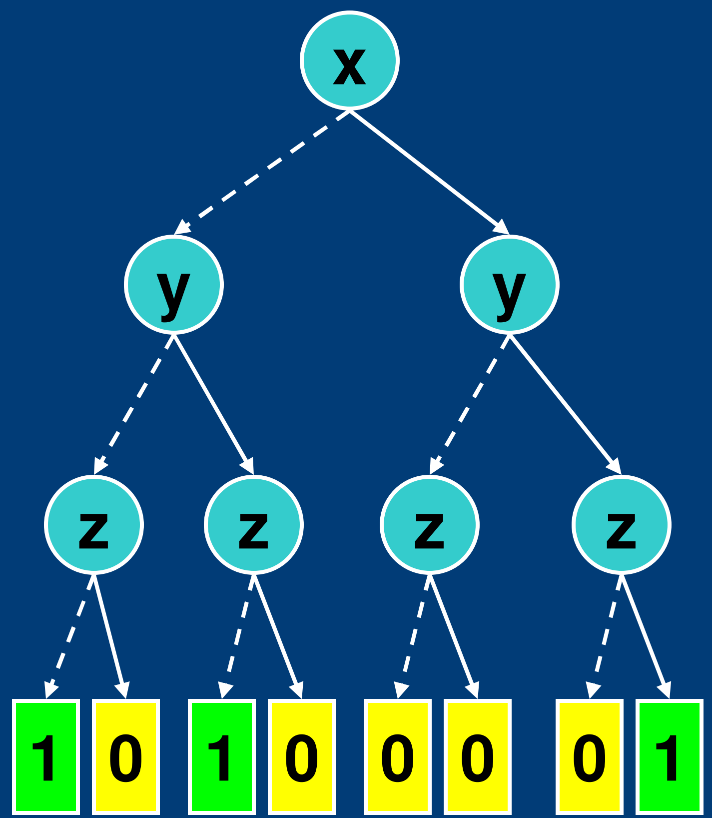
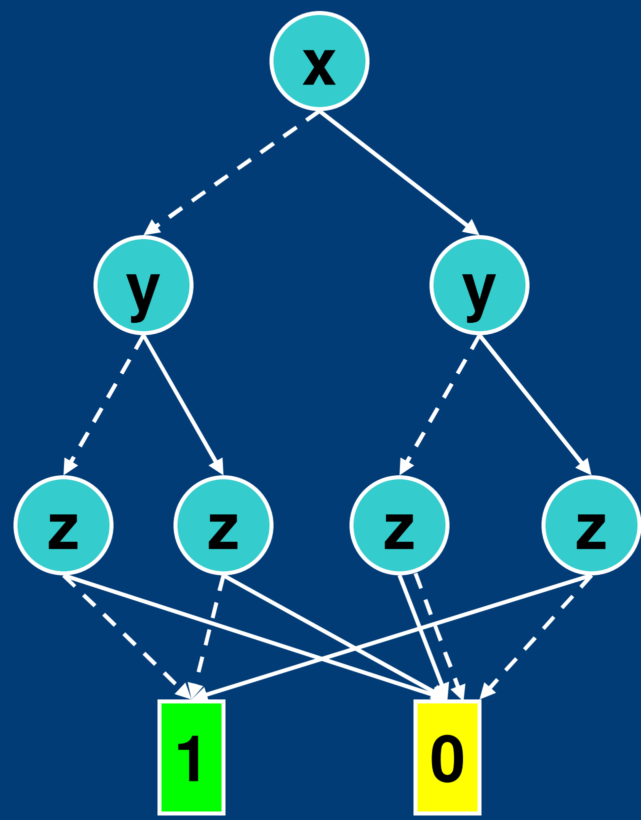
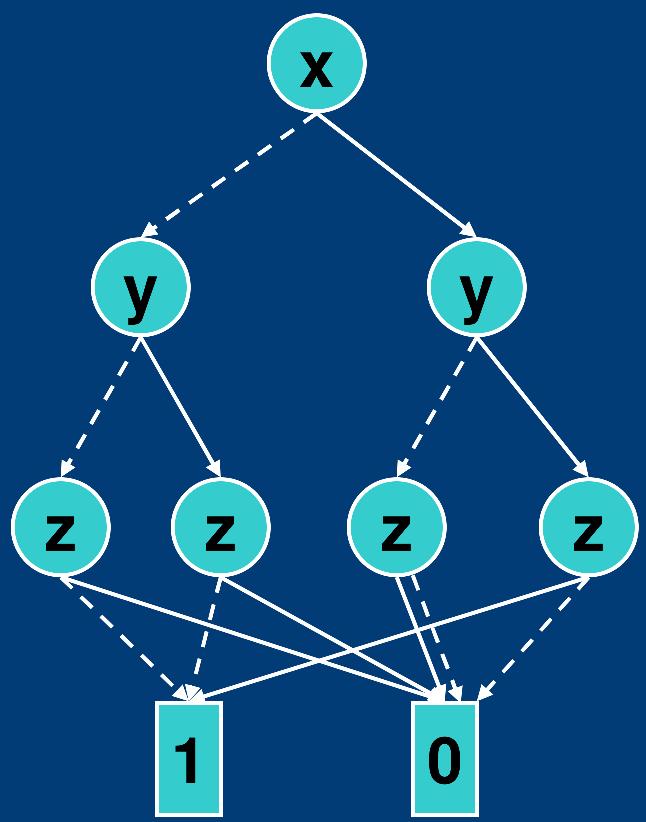




Deriving BDDs and ZDDs

- Rule 1 merges terminal vertexes
- Rule 2 reuses common sub-graphs



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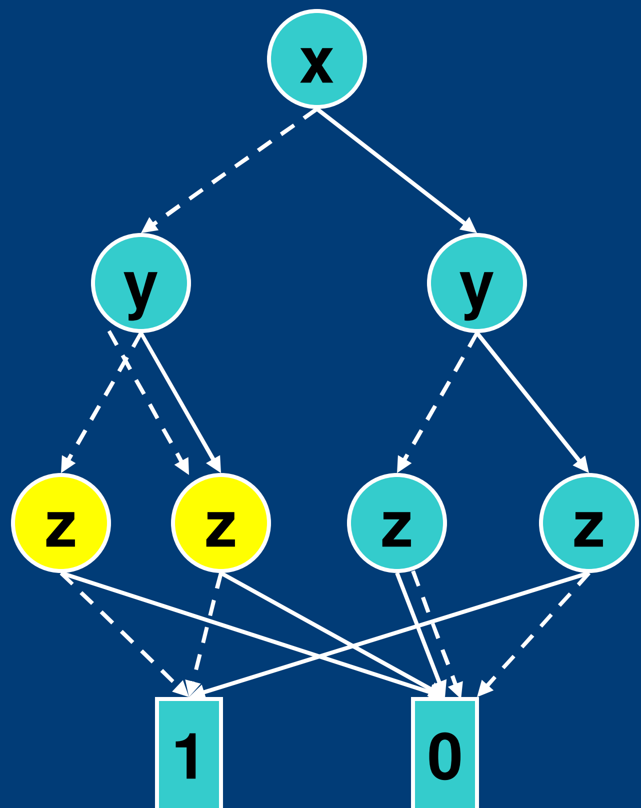
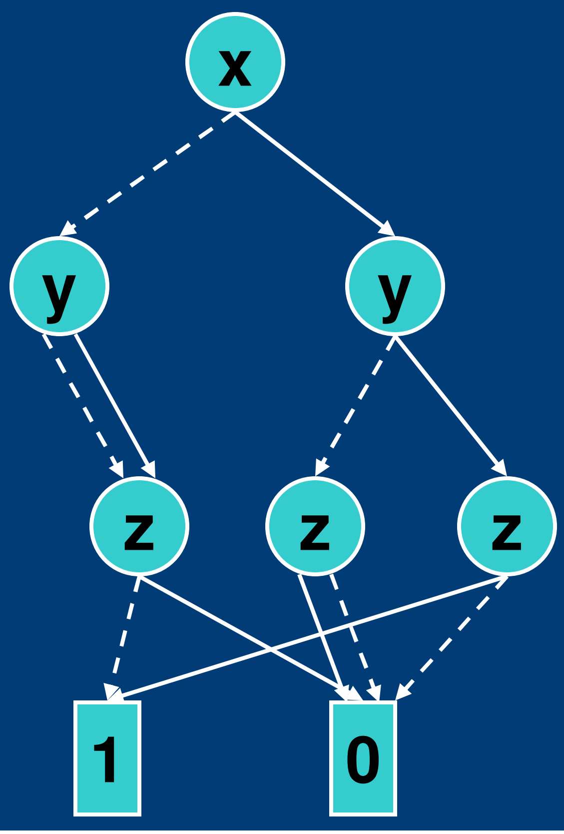


Rule 3 for Deriving BDDs

- Rule 3: Remove "don't care" nonterminal vertexes (whose children are the same)



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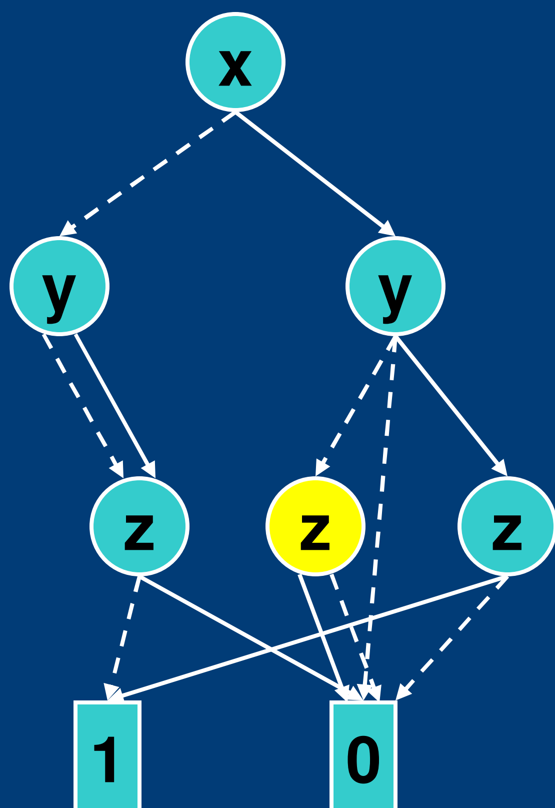

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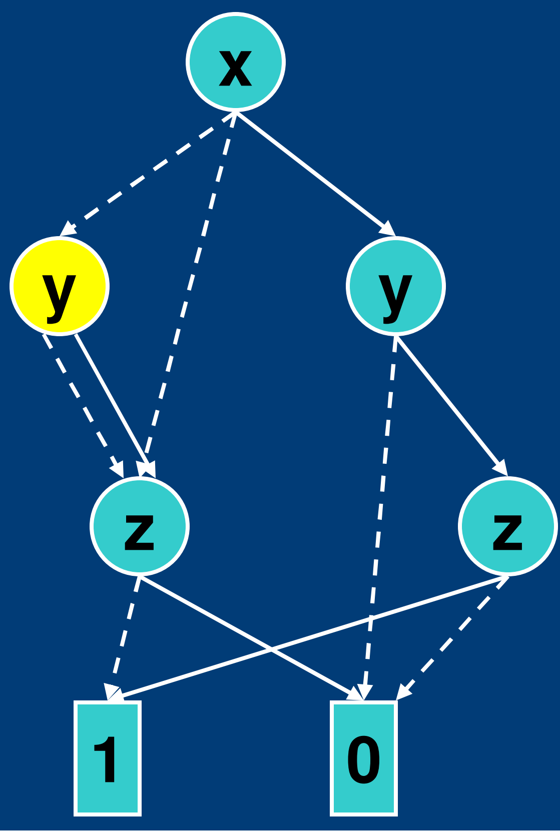
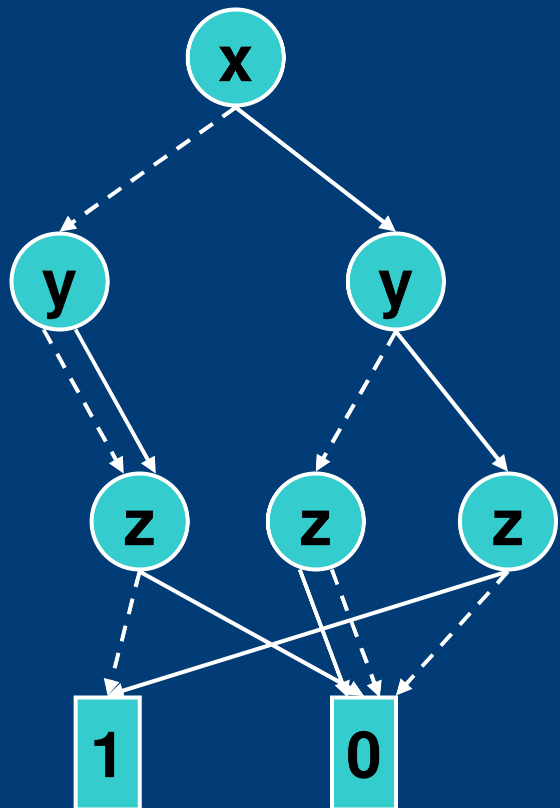


Rule 3 for Deriving ZDDs

- Rule 3: Remove nonterminal vertexes whose 1-edge points to the terminal vertex labeled by 0



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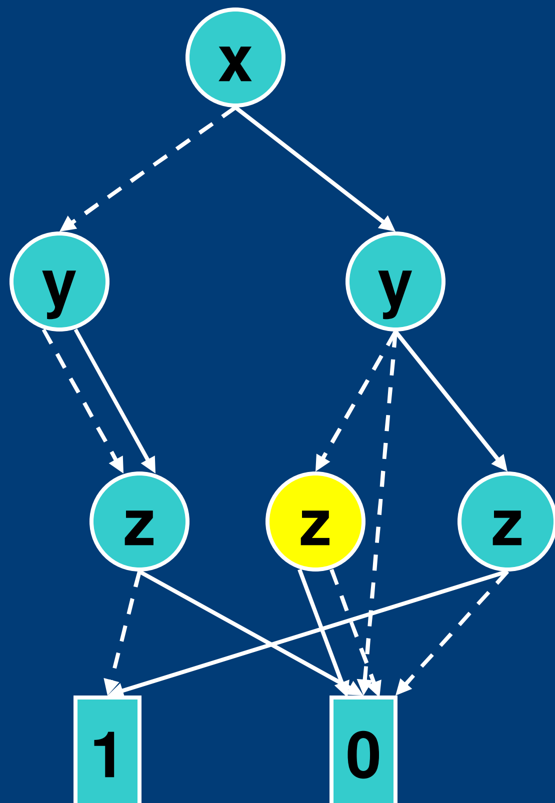

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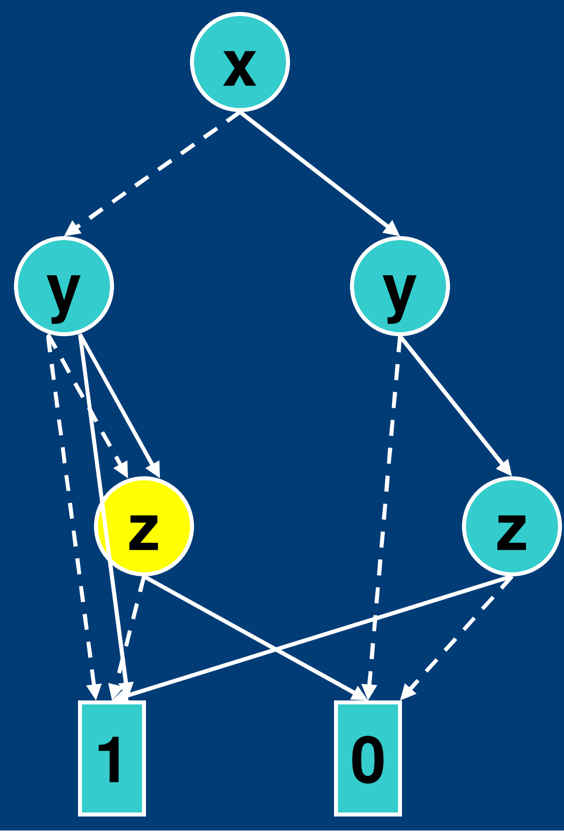
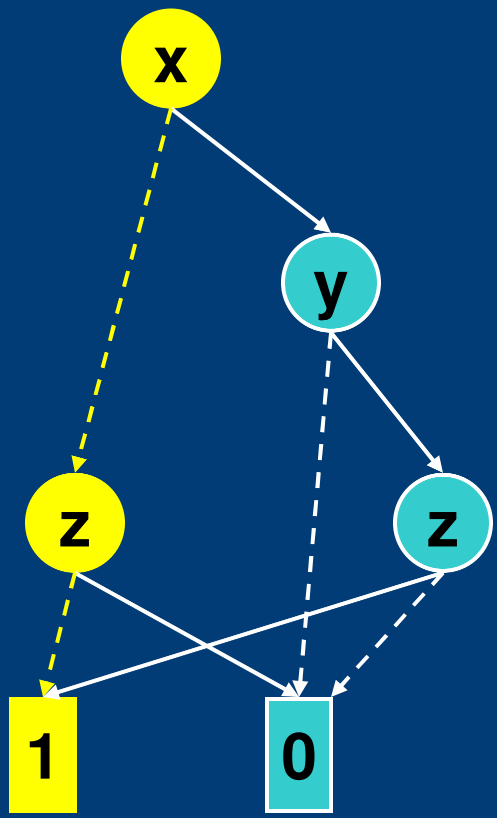
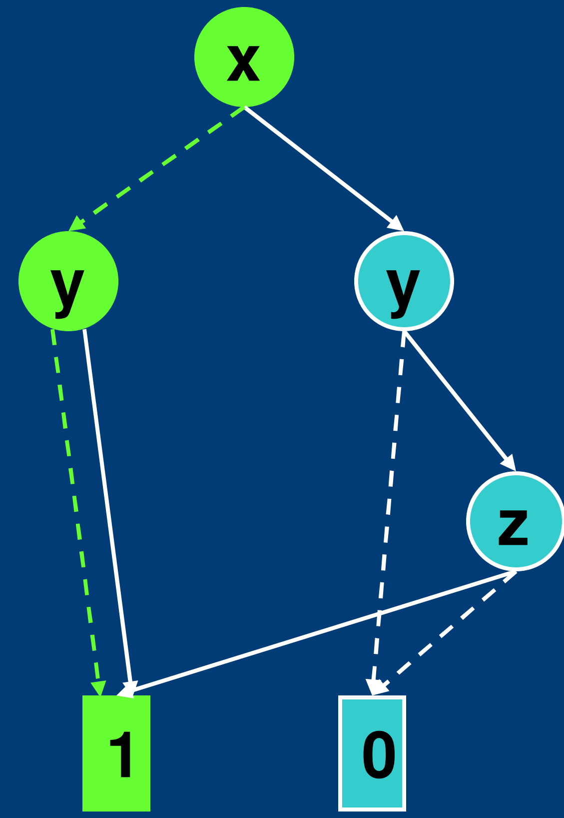


Resulting BDD



Along a path, missing variables may be assigned to either 1 or 0

Resulting ZDD



Along a path, missing variables are assigned to 0

Ordered BDDs/ZDDs

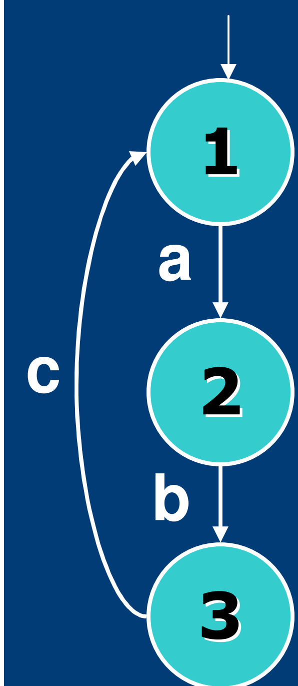
- Boolean variables are ordered
- For any vertex p and either of its nonterminal children q: $var(p) > var(q)$ holds
- Given an order, BDDs/ZDDs for Boolean functions are canonical
- Ordering can have a significant impact on efficiency
  - Will describe an ordering heuristic later

To Apply BDDs/ZDDs in FSV

- Encode FSAs as Boolean functions represented as BDDs/ZDDs
- Compute reachable states with BDDs/ZDDs
  - BDD-based: standard algorithm
  - ZDD-based: modified algorithm

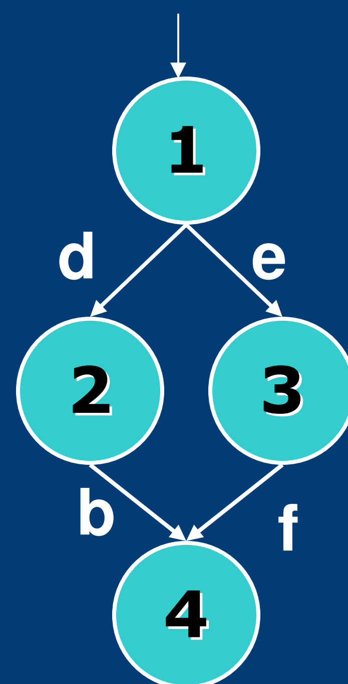
Encoding FSA States

- A process FSA with *n* states can be encoded with $\lceil \log_2 n \rceil$ Boolean variables
- Each process FSA uses a different set of variables



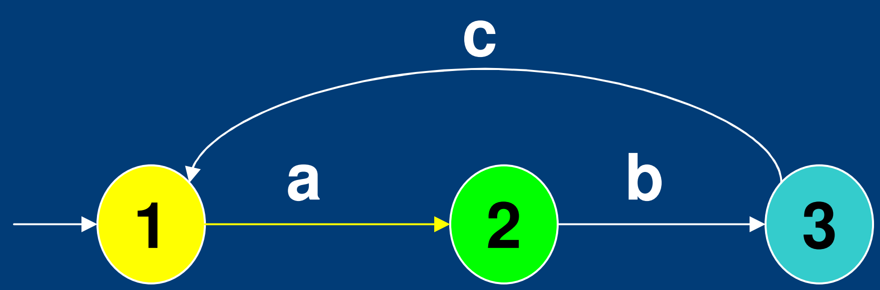
State 1 : $(\neg x_1 \wedge \neg x_2)$
State 2 : $(\ \ x_1 \wedge \neg x_2)$
State 3 : $(\neg x_1 \wedge \ \ x_2)$

State 1 : $(\neg x_3 \wedge \neg x_4)$
State 2 : $(\ \ x_3 \wedge \neg x_4)$
State 3 : $(\neg x_3 \wedge \ \ x_4)$
State 4 : $(\neg x_3 \wedge \neg x_4)$



Encoding Transitions

- Uses two sets of paired variables
  - Source variables: $x_1, x_2, \ldots$
  - Destination variables: $x_1', x_2', \ldots$



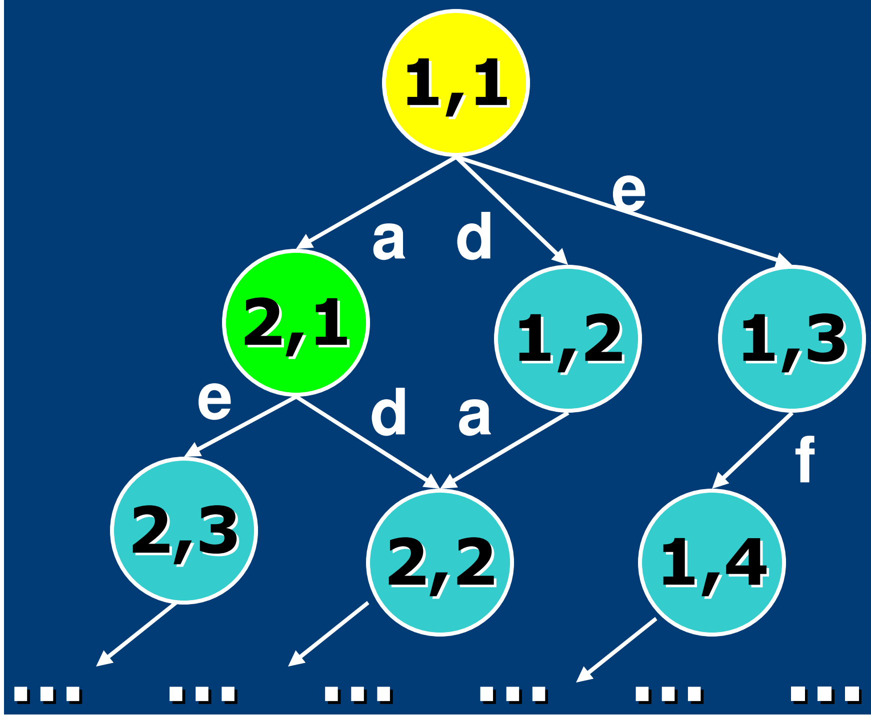
Transition 1→2

$$(\neg x_1 \wedge \neg x_2) \wedge$$
$$(x_1' \wedge \neg x_2')$$

- Transitions labeled by the same event are encoded by a single Boolean function
- The whole transition relation is a disjunction of these functions

Encoding Cross Product FSA

- Each state of the cross product FSA is a conjunction of process states



State <1,1> :

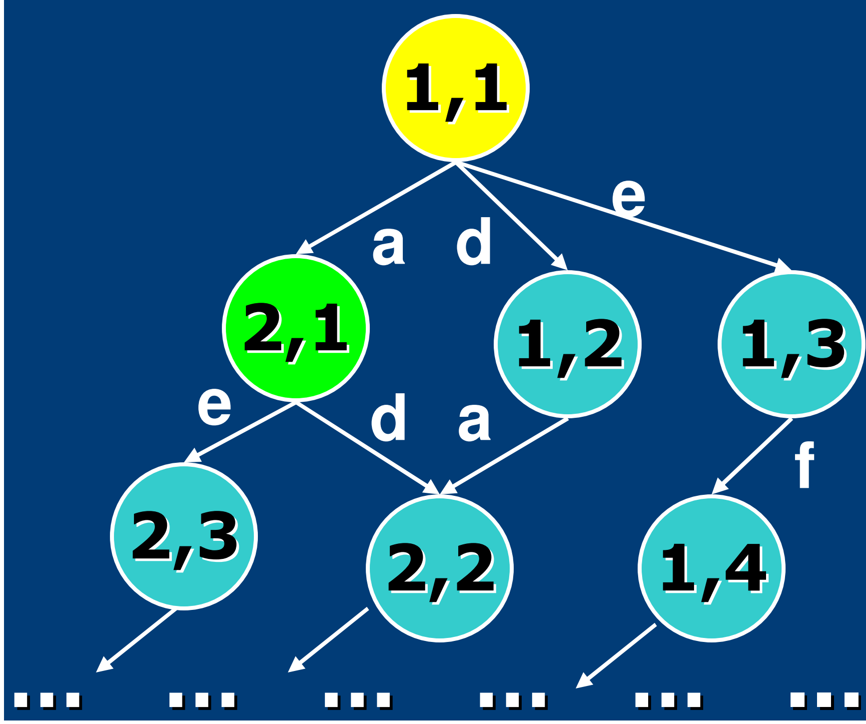
$(\neg x_1 \wedge \neg x_2 \wedge \neg x_3 \wedge \neg x_4)$

State <2,1>

$(x_1 \wedge \neg x_2 \wedge \neg x_3 \wedge \neg x_4)$

Encoding Cross Product FSA

- Each state of the cross product FSA is a conjunction of process states
- The transition is encoded as before



State <1,1> :

$(\neg x_1 \wedge \neg x_2 \wedge \neg x_3 \wedge \neg x_4)$

State <2,1>

$(\ x_1 \wedge \neg x_2 \wedge \neg x_3 \wedge \neg x_4)$

Transition <1,1> → <2,1>

$(\neg x_1 \wedge \neg x_2 \wedge \neg x_3 \wedge \neg x_4) \wedge$

$(\ x_1' \wedge \neg x_2' \wedge \neg x_3' \wedge \neg x_4'\ )$

Computing Reachable States

- Standard fixpoint algorithm: computes *images* until no new states are seen
  - Image: all direct successors for a set of states
- All operations used in the computation can be implemented with BDDs/ZDDs
- Developed a more efficient approach for computing images with ZDDs

Choose a Variable Ordering

- Ordering of variables impacts size of BDDs/ZDDs
- But finding an optimal ordering is NP-hard
- Typical two general approaches
  - Static heuristics to choose a fixed ordering before analysis
  - Dynamic reordering during analysis
- Developed a static ordering heuristic

General Ordering Constraints

- Put each destination variable right after its paired source variable
  - Effective and commonly-used
- Group together Boolean variables encoding each single process FSA
  - States in the same FSA depend more on each other than states from other FSAs

Adapted the FORCE Heuristic

- The FORCE heuristic [Aloul, Markov, Sakallah]: puts variables from FSAs that are *closely related* to each other as close as possible
- We used the number of events shared by two FSAs to measure how closely two FSAs are related
  - The more events shared, the closer they are
  - Within each FSA, variables are ordered arbitrarily

Evaluated Impact of BDDs/ZDDs on FSV

- BDDs/ZDDs implementation
  - Based on the JavaBDD [Whaley] library
  - BDDs and ZDDs are implemented in a consistent way

- Applied to two finite state verifiers:
  - LTSA [Magee, Kramer]:
    Labeled Transition System Analyzer
  - FLAVERS [Dwyer, Clarke, Cobleigh, Naumovich]:
    FLow Analysis for VERification of Systems

Algorithms Evaluated

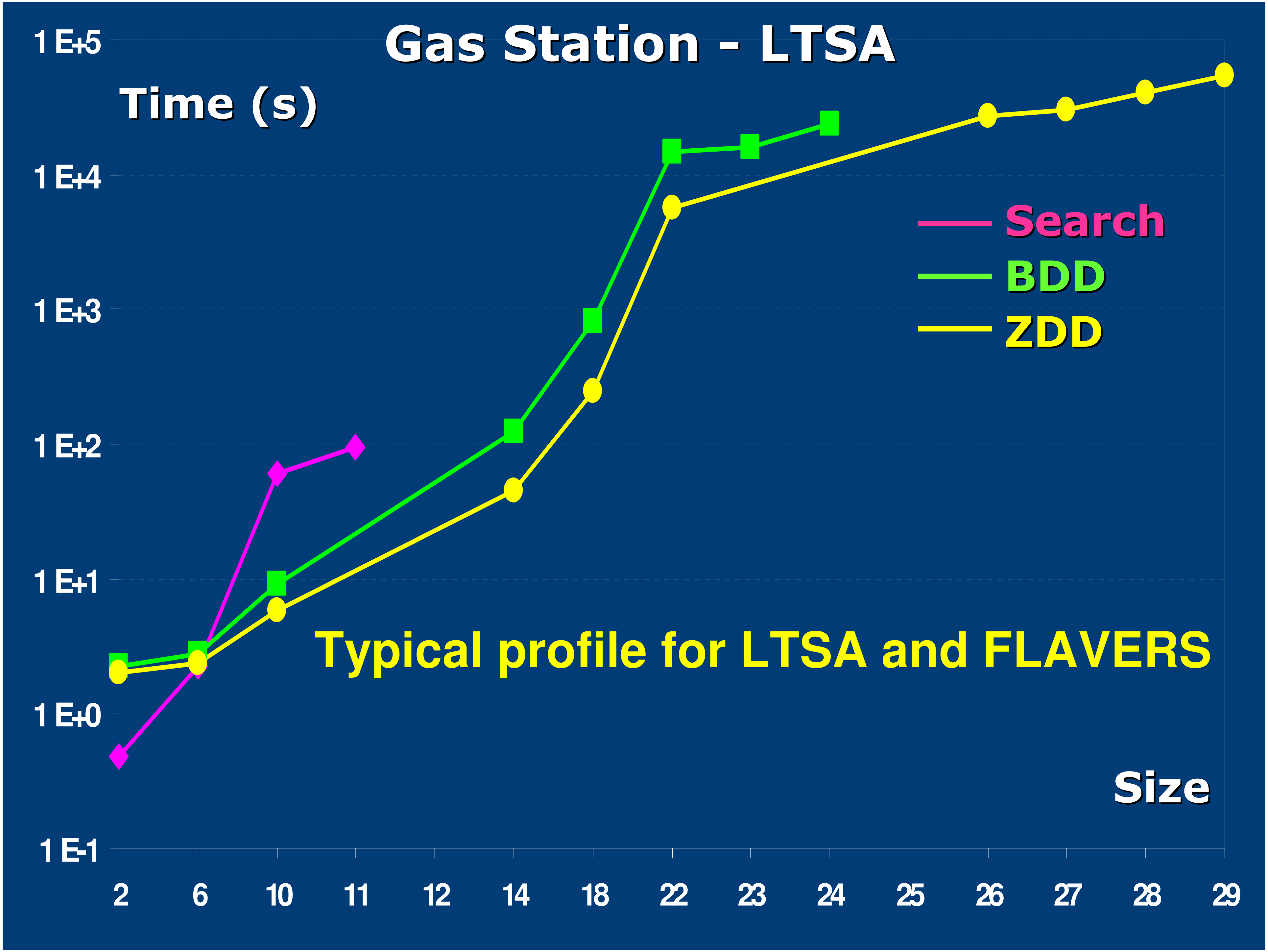
- LTSA
  - The native search algorithm, which constructs a version of the reachability graph and uses a hashtable to store states
  - BDD-based
  - ZDD-based
- FLAVERS
  - The native search algorithm, which uses data flow analysis and a hashtable to store states
  - BDD-based
  - ZDD-based

Systems and Properties

- 13 systems were used
  - Scalable, concurrent
  - 4 systems modeled in LTSA only
  - 4 systems modeled in FLAVERS only
  - 5 systems modeled in both verifiers
- 18 properties in all
- Only considered properties that hold for these systems
  - Ensures that the number of reachable states explored by different algorithms is the same

Metrics

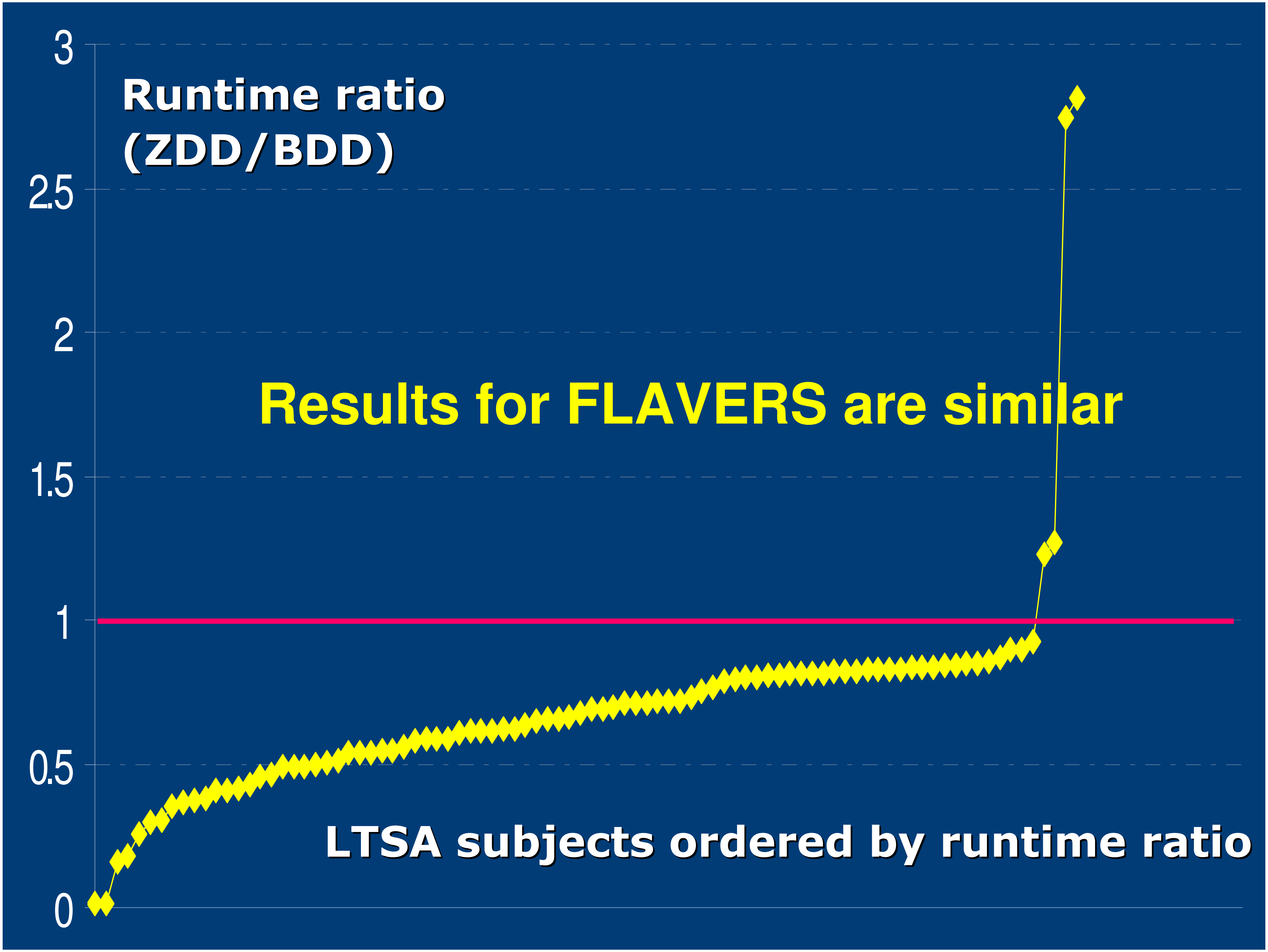
- Ran each algorithm on each system/property, scaling up the size until the algorithm:
  - Ran out of memory, or
  - Ran for more than 24 hours
- Compared
  - ***Runtime***
  - ***The Largest size*** each algorithm could handle for each system



Gas Station - LTSA
Time (s)
1 E+5
1 E+4
1 E+3
1 E+2
1 E+1
1 E+0
1 E-1
Search
BDD
ZDD
Typical profile for LTSA and FLAVERS
Size
2
6
10
11
12
14
18
22
23
24
25
26
27
28
29

Experimental Results

- BDD-based and ZDD-based algorithms could handle much larger systems than both native algorithms in most cases
  - In only one LTSA system, the native algorithm outperformed BDD and ZDD based algorithms
- ZDD-based algorithm could handle larger systems and ran faster than BDD-based algorithm in most cases
  - In only 5 out of 200 subjects, BDD-based outperformed ZDD-based algorithm



3
2.5
2
1.5
1
0.5
0
Runtime ratio (ZDD/BDD)
Results for FLAVERS are similar
LTSA subjects ordered by runtime ratio

Related Work

- BDD-based tools
  - NuSMV [Cimatti, Clarke, Giunchiglia, Roveri]
  - Rulebase [Beer, Ben-David, Eisner, Geist, Gluhovsky, et al]
  - …
- Usage of ZDDs
  - Petri net verification [Yoneda, Hatori, Takahara, Minato]
  - …
- Comparisons between symbolic approach and non-symbolic approach in FSV
  - [Avrunin, Corbett, Dwyer, Pasareanu, Siegel]
  - [Dong, Xu, Ramakrishna, Ramakrishnana, et al]
  - …

Conclusion

- The ZDD-based algorithm was almost always better than the BDD-based and native algorithms
  - Reachable state space is usually sparse
  - ZDDs are good at representing Boolean functions with sparse *supports* (assignments for which the function value is 1)
- More evaluation is needed
  - ZDDs might be useful for other finite state verifiers

Thanks

Questions?

Image Computation w. ZDDs

- The computation needs two ZDDs
  - *S*: represents a set of states using only source variables
  - *T*: represents the whole transition relationship using both the source variables and destination variables
- Missing destination variables in ZDD *S* are assigned to 0, but they should be considered as “don’t care”
  - Treat destination variables in a BDD way
  - Treat source variables in a ZDD way