ABSTRACT

Finite-state verification (FSV) techniques attempt to prove properties about a model of a system by examining all possible behaviors of that model. This approach suffers from the state-explosion problem, where the size of the model or the analysis costs may be exponentially large with respect to the size of the system. Using symbolic data structures to represent subsets of the state space has been shown to usually be an effective optimization approach for hardware verification. The value for software verification, however, is still unclear. In this paper, we investigate applying two symbolic data structures, Binary Decision Diagrams (BDDs) and Zero-suppressed Binary Decision Diagrams (ZDDs), in two FSV tools, LTSA and FLAVERS. We describe an experiment showing that these two symbolic approaches can improve the performance of both FSV tools and are more efficient than two other algorithms that store the state space explicitly. Moreover, the ZDD-based approach often runs faster and can handle larger systems than the BDD-based approach.

Categories and Subject Descriptors
D.2.4 [Software Engineering]: Software/Program Verification—formal methods, model checking, validation

General Terms
Verification

1. INTRODUCTION

Finite-state verification (FSV) approaches attempt to prove properties about a model of a system. These approaches are not as general as theorem-proving, but they are usually easier to use. They do suffer, however, from the so-called state-explosion problem, where the size of the model or the analysis costs may be exponentially large with respect to the size of the system being analyzed.

There are a number of different techniques for dealing with this state-explosion problem. Using symbolic data structures such as Binary Decision Diagrams (BDDs) to represent subsets of the state space [8, 15] has been shown to usually be an effective optimization approach for hardware verification. The value for software verification, however, is still unclear [2, 11].

In this paper we report on our exploration of two symbolic data structures, BDDs and Zero-suppressed Binary Decision Diagrams (ZDDs), applied to two different FSV tools, LTSA [14] and FLAVERS [12]. We describe how the systems to be verified are encoded as Boolean functions and how the verification problem is solved by Boolean operations. Moreover, we present a BDD-based algorithm and a novel ZDD-based algorithm for implementing a key Boolean operation used in the verification. We demonstrate that these two symbolic approaches can improve the performance of both FSV tools and are more efficient than the tools’ native algorithms that store the state space explicitly. In addition, we find that on average, the ZDD-based approach uses only 60% as much time as the BDD-based approach in LTSA and 71% in FLAVERS, and can often handle larger systems than the BDD-based approach. Based on these experimental results, we believe that it would be worthwhile to explore applying this ZDD approach to other FSV techniques.

In the next two sections of this paper, we provide an overview of LTSA and FLAVERS so that the reader can understand how the verification problem is encoded using Boolean functions and how the verification problem is solved via the Boolean operations. Moreover, we present a BDD-based algorithm and a novel ZDD-based algorithm for implementing a key Boolean operation used in the verification. We demonstrate that these two symbolic approaches can improve the performance of both FSV tools and are more efficient than the tools’ native algorithms that store the state space explicitly. In addition, we find that on average, the ZDD-based approach uses only 60% as much time as the BDD-based approach in LTSA and 71% in FLAVERS, and can often handle larger systems than the BDD-based approach. Based on these experimental results, we believe that it would be worthwhile to explore applying this ZDD approach to other FSV techniques.

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2. LTSA OVERVIEW

The Labeled Transition System Analyser (LTSA) [14] is an FSV tool for modeling and analyzing the behavior of systems represented by labeled transition systems. In LTSA, a system is modeled by a set of interacting processes, where each process is described in the Finite Process Language. Systems represented by labeled transition systems. In LTSA, a system is modeled by a set of interacting processes, where each process is described in the Finite Process Language. Each process is represented by a labeled transition system. In LTSA, a system is modeled by a set of interacting processes, where each process is described in the Finite Process Language. Each process is represented by a labeled transition system.

A version of FLAVERS that handles Java programs is under development.
with the final node. Tuples in this set that contain any non-accepting state of any constraint FSA are excluded from this check. After excluding these, if there is a tuple including a non-accepting property FSA state, then FLAVERS returns inconclusive, meaning that the property may be violated. (Since the model may overapproximate the behavior of the actual system, the property might be violated in the model, but not in the actual system.) Otherwise, FLAVERS returns conclusive. The current implementation of FLAVERS does not check for deadlock.

The state propagation algorithm constructs and stores all the tuples associated with each node of the TFG. As with LTS, these tuples are stored in a hash table, but as mentioned above, this is not suitable for very large systems. Considerable research has been devoted to finding alternative data structures. One promising direction, as described in the following sections, is to use a symbolic data structure to replace the hash table.

The state propagation algorithm can be thought of as exploring the cross-product of the TFG, the property FSA, and all constraint FSAs, where each state in the cross-product FSA is a node-tuple pair. From this point of view, the basic approach of FLAVERS is similar to that of LTS, although the detailed implementation is quite different. We therefore focus on LTS as we explain how the symbolic data structures are applied in the following sections.

4. APPLYING BDDS

In this section, we give a brief introduction to BDDs and then describe their application in FSV using LTS.

4.1 Background on BDDs

A BDD [7] is a directed acyclic graph that can represent a Boolean function. BDD is derived from the Binary Decision Tree (BDT), which can also represent Boolean functions. Figure 2(a) shows an example of a BDT representation.

A BDT representing a Boolean function comprises two kinds of vertexes, nonterminal vertexes and terminal vertexes, as denoted in Figure 2 (a) by the circular and rectangular nodes, respectively. Each nonterminal vertex p has a labeled Boolean variable var(p), and two children, low(p) and high(p). Child low(p) is connected by the low-edge of p, shown as a dashed line in Figure 2 (a), and child high(p) is connected by the high-edge of p, shown as a solid line in Figure 2 (a). Each terminal vertex is labeled by either 0 or 1. Depending on its label, a terminal vertex is called a 0-terminal or 1-terminal vertex.

A Boolean function composed of n Boolean variables has 2^n assignments. We call the assignments under which the function value is one 1-assignments, while assignments making the function zero are 0-assignments. A BDD representation for the function has 2^n paths, where each path corresponds to an assignment of the function. In each of these paths, there are n nonterminal vertexes, each of which is labeled by a different Boolean variable. In the corresponding assignment of the path, the value of variable v is determined by which edge of the nonterminal vertex labeling v is in the path: the low-edge means that v = 0 whereas the high-edge means v = 1. The label of the terminal vertex in a path indicates the value of the function under the corresponding assignment. In Figure 2 (a), for example, the path “1 → 2 → 5 → 11” represents the assignment f(x = 0, y = 1, z = 1) = 0.

Usually, a total ordering < is put over all Boolean variables in a BDD. This means that for any nonterminal vertex p and q, if q is a child of p, then var(p) < var(q) must hold. In Figure 2 (a), for example, the variables are ordered as x < y < z.

A BDD is derived from a BDT by applying the following three reduction rules: Rule 1 merges all 0-terminal vertexes and merges all 1-terminal vertexes. Rule 2 shares common sub-graphs. Specifically, when two nonterminal vertexes are the same in that they have the same labeled variable and the same children, then one of them can be eliminated and all its incoming edges redirected to the other one. Rule 3 removes “don’t care” nonterminal vertexes. A “don’t care” nonterminal vertex is one whose two children are the same. When such a vertex is eliminated, its incoming edges are redirected to its child. The basic idea of Rule 3 is that variables that do not affect the functional assignments do not need to be stored in the BDD representation. Given a BDT, these three rules are applied recursively from the bottom-up. The final resulting BDD is called an ordered reduced BDD. From now on, all BDDs we discuss will be ordered reduced BDDs. Figure 2 (b) shows an example of the BDD representation derived from the BDT shown in Figure 2 (a).

Note that the variable ordering selected for a BDD representation is very important to the efficiency of the representation [7]. To find the optimal ordering for a BDD representation, however, is an NP-complete problem [6]. We discuss the heuristic used in our work to find a good ordering in Section 6.

4.2 Using BDDs with LTS

A BDD is just a data structure for representing Boolean functions. Thus, to use BDDs in FSV, the verification problem must be encoded using Boolean functions. More specifically, the following problems need to be addressed: how to encode FSA states and transitions; how to compute the transition functions; and how to check for safety property violation and deadlock, by using Boolean functions. We now describe the approaches we used for each of these.

- **Encoding the FSA States.** Given an FSA with n states (not counting the ERROR state in a process FSA but counting the VIOLATION state in the property FSA), we use \(\log_2 n\) Boolean variables to encode these states. Each state is represented by a Boolean function that has only one 1-assignment. For example, for the client FSA shown in Figure 1 (a), 2 Boolean variables, \(x_1\) and \(x_2\), are enough to
encode the 3 states in the FSA. State 1 in this FSA can be represented by the Boolean function \( f_{1,i} = \neg x_1 \land \neg x_2 \), which can be 1 only when \( x_1 = 0 \) and \( x_2 = 0 \).

After the states in each FSA are encoded, the composite state \( s = (s_1, s_2, \ldots, s_n) \) is encoded by \( f_s = \bigwedge_{1 \leq i \leq n} F_{i,s_i} \), where \( F_{i,s_i} \) is the Boolean function for state \( s_i \) in \( F_i \). For a set of FSA states, the Boolean function \( f_s \) is simply the disjunction of Boolean functions for each state in the set.

**Encoding the Transitions.** A transition has three components: a source state, a destination state, and an event. To encode a transition, two sets of Boolean variables are used in order to distinguish the source state and the destination state. The event is not encoded explicitly using Boolean variables. Instead, transitions triggered by different events are represented by different Boolean functions.

The two sets of variables used in encoding transitions are \( X' \), the source variable set used for encoding source states, and \( X'' \), the destination variable set used for encoding destination states. Each variable \( x \in X' \) has one and only one paired variable \( x' \in X' \) and vice versa.

The Boolean function for a transition from \( s \) to \( s' \) is simply \( f_s \land f_{s'}^c \), where \( f_s \) is the Boolean function for state \( s \) using the source variables and \( f_{s'}^c \) the Boolean function for state \( s' \) using the destination variables. For transition \( \Delta((s_1, s_2, \ldots, s_n), e) = (s'_1, s'_2, \ldots, s'_n) \) in a composite FSA where \( s'_i = \delta_i(s_i, e) \), its Boolean function is \( F_{\Delta(s_1, s_2, \ldots, s_n, e)} = [\bigwedge_{1 \leq i \leq n} F_{i,s_i}] \land [\bigwedge_{1 \leq i \leq n} F_{i,s'_i}] = \bigwedge_{1 \leq i \leq n} (F_{i,s_i} \land F_{i,s'_i}) \), where \( F_{i,s_i} \) is the Boolean function encoding state \( s_i \) using source variables and \( F_{i,s'_i} \) is the Boolean function encoding state \( s'_i \) using destination variables.

The Boolean function for all transitions in a composite FSA composed of \( n \) FSAs is built in three steps. First, a Boolean function for all transitions triggered by event \( e \) in each FSA \( F_i \) is constructed. This function is denoted by \( F_e \). Second, the Boolean function \( F_e \) is \( \bigwedge_{1 \leq i \leq n} F_{e,s_i} \) is built to encode all transitions in the composite FSA that are triggered by event \( e \). Third, the Boolean function for all transitions in the composite FSA \( F = \bigvee_{e \in \Sigma} F_e \). Note that which transitions are fired by which event cannot be distinguished using the function \( F_e \), but that information is not needed for checking for violations of safety properties and for deadlock, as shown below.

**Computing the Transition Functions.** A key step in exploring reachable composite states is to compute all states reachable by transitions starting from a given source state. With states and transitions encoded in Boolean functions, these three operations can be implemented efficiently [7].

When BDDs are used as the data structure for representing Boolean functions, these three operations can be implemented efficiently [7]. Furthermore, as mentioned in [15], the and operation and the existential quantification operation can be combined together into the andExists operation, which can often improve the performance substantially because it avoids creating the BDD representing \( f_s^c \). In addition, we notice that the replace operation can also be combined into the andExists operations, further eliminating the temporary BDD for \( f_s^{[2]} \). We give this BDD algorithm in Figure 3.

The algorithm in Figure 3 is a recursive procedure. Lines 1 to 3 compute the base cases. Lines 7 to 19 carry out the and operation. At line 20, when \( v \) is a destination variable, the replace operation is computed. In lines 24 and 25, when \( v \) is a source variable, the existential quantification operation is computed. Note that this algorithm computes destination states for a set of source states at the same time.

To ensure that the resulting BDD is still ordered, a variable ordering restriction is imposed. This requires that each destination variable immediately follows its paired source variable. Without this restriction, the steps at lines 21 to 23 may break the variable ordering. This restriction is often used in other BDD-based FSV tools.

**Checking for Violation of Safety Properties and for Deadlock.** To check for violation of safety properties, a Boolean function, denoted by \( V \), is created to represent all VIOLATION states from each property FSA. Note that variables used for encoding process FSA are not included in \( V \).

To check whether a composite state \( s \) violates a property, one can compute \( f_s \land V \) and then check whether the result is equal to \( \bot \), the function with no 1-assignments. If \( f_s \land V \neq \bot \), it means that the state \( s \) contains no VIOLATION states and thus no violation will be reported. Otherwise a violation is found.

For the deadlock check, another Boolean function, denoted by \( D \), is created to represent all composite states at which no deadlock happens. This function is built in a way similar to the function that encodes all transitions. For a system with \( n \) FSAs, one first identifies all states in each FSA \( F_i \) such that these states can go to a non-ERROR states by a transition triggered by event \( e \). Let \( f_i(e) \) denote the function representing these states in \( F_i \). Then, the Boolean function \( f(e) = \bigwedge_{1 \leq i \leq n} f_i(e) \) is built to encode all composite states that can go to a non-ERROR composite state by a transition fired by \( e \). At last, \( D = \bigvee_{f \subseteq \Sigma} f(e) \), which represents all composite states at which no deadlock happens, since they all can go to at least one non-ERROR composite state by some transition.

To check whether deadlock occurs at a composite state \( s \), one can simply compute \( f_s \land D \). If the result is equal to \( f_s \), it means that state \( s \) is one of the states represented by \( D \) and thus no deadlock happens at state \( s \). Otherwise a deadlock is reported.

There are two differences between FLAVERS and LTSA at this point. First, as mentioned above, FLAVERS does not check for deadlock. Second, when checking a property violation, FLAVERS considers every element included in a node-tuple. Indeed, the function \( V \) in FLAVERS represents all node-tuples such that the node is the final node of the TFG, the property state in the tuple is non-accepting and
//Given a BDD p encoding a set of source states and
//BDD t encoding all transitions of the system,
//return another BDD r encoding all destination states
//of source states represented by p
//A computed table is used to store results of
//previous computations

APPLYTRAN(p, t) {
    if p is the 0-terminal vertex, return 0-terminal vertex
    if t is the 0-terminal vertex, return 0-terminal vertex
    if t is the 1-terminal vertex, return 1-terminal vertex
    if the computed table contains the entry {p, t},
    return the stored result
    end if
    
    //The following steps carry out the and operation
    if p is the 1-terminal vertex or var(p) > var(t)
        let r0 = APPLYTRAN(p, low(t))
        let r1 = APPLYTRAN(p, high(t))
        let v = var(t)
    else if var(p) < var(t)
        let r0 = APPLYTRAN(low(p), t)
        let r1 = APPLYTRAN(high(p), t)
        let v = var(p)
    end if
    
    if v ∈ X'
        //If v is a destination variable,
        //do the replace operation
        let v2 ∈ X be the paired variable of v
        find or make a BDD node r such that:
        var(r) = v2 and low(r) = r0 and high(r) = r1
    else
        //Else v is a source variable,
        //do the existential quantification operation
        let r = or(r0, r1)
    end if
    
    put {(p, t), r} into the computed table
    return r
}

Figure 3: An algorithm that computes transition functions using BDDs

Build the following BDDs:

\( f_p \): represents the composite start state
\( \mathcal{F} \): represents all transitions in the system
\( V \): represents all VIOLATION states in property FSAs
\( D \): represents all composite states with no deadlock

Let \( f_{\text{visited}} \) be the BDD representing all states that have been encountered
\( f_{\text{visited}} = f_p \)

while true
    \( f_1 = APPLYTRAN(f_{\text{visited}}, \mathcal{F}) \)
    if \( f_1 \land \neg V \neq \bot \), return Property Violated
    if \( f_1 \land \neg D \neq f_1 \), return Deadlock
    if \( f_1 == f_{\text{visited}} \), return No Violation and no Deadlock
    \( f_{\text{visited}} = f_1 \lor f_{\text{visited}} \)
end while

Figure 4: An algorithm that checks safety properties and deadlock using BDDs

5. APPLYING ZDDS

We now discuss how a ZDD [16] is used to represent a Boolean function, focusing on the differences between a ZDD and BDD representation. We also describe the application of ZDDs in the context of LTSA.

5.1 Background on ZDDS

A ZDD, like a BDD, can represent a Boolean function and is also derived from a BDT. The fundamental difference between a ZDD representation and a BDD representation lies in the difference in the third reduction rule. Rule 3' for deriving ZDD representations removes a nonterminal vertex \( p \) when \( high(p) \) is a 0-terminal vertex. After this vertex is removed, all its incoming edges are redirected to \( low(p) \).

Assignments with no corresponding paths in the ZDD must be stored. Basically, each path in a ZDD represents a functional assignment in the same way as the paths in a BDD or BDD do. Not every functional assignment, however, has a corresponding path in its ZDD representation. Assignments with no corresponding paths in the ZDD must be 0-assignments. Moreover, not every variable is included in a path of a ZDD. Missing variables in a BDD representation, called “don’t care” variables, may have value 0 or 1 in the corresponding assignments. In contrast, missing variables in a ZDD representation must be assigned to 0 in the corresponding assignment under the ZDD semantics. In the example shown in Figure 2 (c), there is no path corresponding to the 0-assignment \( f(x = 1, y = 0, z = 1) = 0 \).

For the path “1 \( \rightarrow \) 3 \( \rightarrow \) 9” in this figure, variable \( z \) is missing in the path, so the path actually represents the assignment \( f(x = 1, y = 0, z = 0) = 0 \).

To see why a ZDD represents the Boolean function represented by the BDT from which the ZDD is derived, we need only look at Rule 3’, since Rule 1 and Rule 2 do not change any path in the graph and thus do not change the functional assignments. Consider all paths in the graph that pass through nonterminal vertex \( p \) where \( high(p) \) is the 0-terminal vertex. These paths include either the high-edge of \( p \) or the low-edge of \( p \). In the former case, these paths must reach the 0-terminal vertex, which means that they represent 0-assignments. Therefore, the high-edge of \( p \) can be
removed according to the rationale described above. In the latter case, no matter which terminal vertex these paths end with, the variable \( \text{var}(p) \) must be assigned to 0 in the corresponding assignments. Therefore, the low-edge of node \( p \) can also be removed. As a result, vertex \( p \) can be eliminated from the graph, as defined in Rule 3.

Like BDDs, the ZDDs we consider are also ordered and reduced. Again, the variable ordering still plays a critical role in affecting the efficiency of the ZDD representation.

5.2 Using ZDDs with LTSA

To use ZDDs to check for safety property violations and for deadlock, one needs to solve the same problems as for BDDs. Some of the solutions to these problems for ZDDs, including how to encode states and transitions and how to check for deadlock, are the same as those for BDDs and are not repeated here. We describe below the methods for ZDDs that are different from those for BDDs, namely, how to compute transition functions and how to check for safety property violations.

- Computing the Transition Functions. The algorithm described in Figure 3 that uses Boolean functions and BDDs to compute transition functions is certainly applicable for ZDDs, although the underlying data structure is changed from BDDs to ZDDs. To use that algorithm with ZDDs, however, one must take into account the different semantics of the BDD and ZDD representations. Specifically, as shown in Figure 3, when computing the and operation between \( p \) and \( t \) using ZDD representations, note that \( p \) contains variables from \( X \)' only, whereas \( t \) contains variables from both \( X \) and \( X' \). Under the semantics of ZDDs, missing variables in a ZDD mean that those variables are considered to have value 0 in their functional assignments. As a result, simply computing \( p \land t \) using their ZDDs does not get the right answer. A naive approach for solving this problem is to add variables from \( X' \) to the ZDD \( p \). This approach is plausible but not efficient, because it increases the size of the ZDDs and thus degrades performance. A better solution is to design a new operation that treats the ZDD \( p \) differently. The idea is that the set of variables defining a Boolean function forms the domain set of the function. In a ZDD representation for a Boolean function, those missing variables are considered to have value 0 only when they belong to the domain of the Boolean function. Variables not in this domain set, although they are definitely missing in the ZDD, should be considered as "don't care" variables. In our case, the set \( X \) is the domain of \( p \) and variables from \( X' \) are treated as "don't care" variables.

Accordingly, we can now change the algorithm shown in Figure 3 so that it can be used with ZDDs. Figure 5 gives the adapted algorithm. Lines 1 to 5 of this figure compute the base cases of the recursive procedure. In lines 9 to 21, the procedure handles the case where \( t \) is the 1-terminal vertex or \( \text{var}(t) \in X \), whereas the case where \( \text{var}(t) \in X' \) is handled by lines 22 to 29.

For the case where \( t \) is the 1-terminal vertex or \( \text{var}(t) \in X' \), the procedure computes the and operation between \( p \) and \( t \), and then computes the existential quantification operation. There are three sub-cases. First, when \( t \) is the 1-terminal vertex or when \( \text{var}(p) < \text{var}(t) \) (lines 10 to 12), only the low-edge of \( p \) needs to be considered. This is because for all paths including the high-edge of \( p \), their corresponding assignments must assign \( \text{var}(p) \) to 1. On the other hand, \( \text{var}(p) \) does not appear in \( t \), which means that \( \text{var}(p) \) is always 0 in all assignments represented by \( t \). Therefore, the intersection between assignments represented by \( t \) and those represented by \( p \) including the high-edge of \( p \) is simply empty. Also because of this, the existential quantification operation ignores the high-edge of \( p \). The second sub-case (lines 13 to 15) is symmetric to the first one. In the third sub-case (lines 16 to 21), both children of \( p \) should be considered.

When \( \text{var}(t) \in X' \), the procedure first computes the and operation (lines 24 and 25) and then the replace operation (lines 26 and 27). Note that \( \text{var}(t) \) in this case is a variable from \( X' \) and is a "don't care" variable in \( p \). Therefore, both the high-edge and the low-edge of \( t \) need to be considered.

As in the BDD algorithm in Figure 3, this ZDD algorithm also requires each destination variable to be ordered immediately after its paired source variable.

- Checking for Safety Property Violations. The approach to checking for a safety property violation with ZDDs is essentially similar to the one used with BDDs. The difference here again involves the missing variables. Like the method used with BDD, the function \( V \) is first built to encode all VIOLATION states. This ZDD representation for this function contains only variables used for encoding property FSAs. Then, \( f_s \land V \) is computed with ZDDs. Here, variables used for encoding process FSAs should be treated as "don't care" variables with respect to the ZDD representing \( V \), just like how destination variables are handled when computing the transition functions.

Note that this problem does not exist in FLAVERS. As explained before, the function \( V \) in FLAVERS includes information for every element of a node-tuple. Thus, the domain set of function \( V \) includes all source variables.

To check for violation of safety properties or for deadlock with ZDDs, we can simply reuse the algorithm shown in Figure 4. The performance difference between the BDD-based algorithm and the ZDD-based one comes mainly from the compactness difference between these two representations.

6. EXPERIMENTAL METHODOLOGY

We evaluated six different algorithms in our experiment. For LTSA, we compared the native search algorithm of the tool and the BDD-based and ZDD-based algorithms. For FLAVERS, we compared the native state propagation algorithm and the BDD-based and ZDD-based algorithms. We implemented the BDD-based and ZDD-based algorithms using the JavaBDD [13] package. We built a lightweight BDD/ZDD package from the JavaBDD package, dropping some features such as variable reordering. In addition, we implemented this package in a consistent way for BDDs and ZDDs in that both diagrams use the same hash function, the same cache mechanism, the same memory management strategy and the same data structure for representing BDD/ZDD nodes.

As mentioned before, the variable ordering is critical to the efficiency of both representations. Roughly speaking, there are two approaches to finding a good ordering. A static approach uses heuristics to choose an ordering that is maintained throughout the analysis. Dynamic approaches, on the other hand, modify the ordering as the various BDDs or ZDDs are constructed. We use a static approach, based on the heuristic called FORCE [1]. The idea of this heuristic is
Figure 5: An algorithm that computes transition functions using ZDDs

7. EXPERIMENTAL RESULTS

All the inputs and results from our experiment are available at http://laser.cs.umass.edu/symbolic.

Table 1 shows the largest size each algorithm can handle for each system under the specified time and memory limits. Also shown in this table is the number of reachable states for the largest size of each system. From this table, we can see that symbolic algorithms can handle much larger systems than non-symbolic algorithms in all except one case. This shows that both symbolic data structures are more efficient than the hash table in both LTSA and FLAVERS.

There are several threats to the validity of our results. First, the selection of example systems may bias the results. Most of our systems represent somewhat unrealistic programs that have been constructed to illustrate issues in the design of concurrent systems. These examples may not adequately represent the range of systems to which the FSV tools might be applied. A second threat arises from our restriction to systems for which the properties hold. As a consequence, the results reported in the paper may not reflect the performance of these algorithms in cases where the properties are violated. Finally, we only considered one approach to variable ordering for the BDDs and ZDDs. Therefore, it is possible that results inconsistent with ours may be found for some systems with other variable orderings.
The graphs in Figure 6 show the ratio of runtime for the ZDD-based algorithm to the runtime of the BDD-based algorithm for those cases where both algorithms can finish within our time and memory limits. Each point in the graphs gives the ratio for a single subject, a pair consisting of a system at a particular size and a property. The graph on the left shows the ratios for LTSA and the one on the right shows the ratios for FLAVERS. We see that the ZDD-based algorithm almost always runs faster than the BDD-based algorithm. On average, the ZDD-based algorithm uses only 60% as much time as the BDD-based algorithm in LTSA and 71% in FLAVERS.

Figure 7 shows the runtimes for each algorithm used with LTSA for two examples. In Figure 7, the left vertical axis in each graph indicates the runtime information while the right vertical axis indicates the number of states explored. The horizontal axis indicates the size of the given system. The crosses show the number of reachable states at each size of the system, which is independent of the algorithm. Each curve in a graph shows the runtime of an algorithm as the size of the system increases. These curves stop at the largest size the algorithm can handle with the given time and memory limits. The first graph in the figure shows the results for the Santa Claus system [5], which are typical of what we saw with both LTSA and FLAVERS. The native search algorithm is superior for small sizes, but as the size increases, both symbolic algorithms surpass the native search algorithm. The ZDD-based algorithm runs somewhat faster than the BDD-based algorithm and can scale to larger sizes within the time and memory limits we imposed.

The second graph in Figure 7 show the results for version one of the decomposed Chiron system with LTSA, one case in our experiment in which an algorithm that stores individual states was superior to the symbolic algorithm. (This did not occur with FLAVERS.) Examination of this system reveals that it has a relatively small number of reachable states because the processes in the system are highly synchronized.

To see why the ZDD-based algorithm outperformed the BDD-based algorithm, we examined the sizes of the representations in these algorithms. Not surprisingly, the ZDD representations are almost always more compact than the BDD representations. As shown in [16], a ZDD representation tends to be more efficient than a BDD representation for a sparse combination set, where a combination for a given set $S$ is simply a subset of $S$. By letting $S$ be the domain set of a Boolean function, a functional assignment can be represented by a combination of $S$ that includes Boolean variables whose values are 1 under that assignment. Thus, a Boolean function can be represented by a combination set that represents all 1-assignments of the function. Based on this, we may expect ZDDs to be more efficient than BDDs when representing Boolean functions with sparse 1-assignments. In our experiments, the Boolean functions representing reachable composite states usually have a small number of 1-assignments, compared with the number of all possible assignments those functions can have. Whether the distribution of these 1-assignments is actually sparse, however, is not clear. More work regarding the distribution of the 1-assignments of a Boolean function needs to be done.

Our results suggest that the ZDD-based algorithm is the best overall among the algorithms we evaluated in terms of both time and space and it should be the first algorithm to consider when conclusive results are expected.

8. RELATED WORK
There are many BDD-based FSV tools, such as [4,10]. The main difference between our work and these FSV tools is the use of ZDDs.

ZDDs have been used in many domains, such as SAT solvers [9], and arithmetic polynomial manipulation [16]. ZDDs have also been used in verification [17]. In that work, both BDD and ZDD representations are used and compared in model checking CTL properties of Petri nets. Several symbolic approaches using BDDs and ZDDs to manipulate Petri nets are proposed in that paper, and the experimental results suggest that ZDDs are more suitable.

A number of other investigators have compared the performance of different FSV tools, including those that use symbolic algorithms and non-symbolic algorithms [2,11]. Since different tools use different modeling languages to describe the system, there is no guarantee that the models used with different tools are really equivalent. This factor may significantly affect the comparison of different algorithms. In our work, since the algorithms compared are implemented in the same FSV tool, the systems used for comparison are modeled in exactly the same way.
<table>
<thead>
<tr>
<th>Algorithms</th>
<th>LTSA Search</th>
<th>BDD</th>
<th>ZDD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LS</strong> Reachable States</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Token ring</td>
<td>11 6.78 x 10^9</td>
<td>33 4.58 x 10^11</td>
<td>34 2.18 x 10^12</td>
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<tr>
<td>Gas station</td>
<td>11 7.15 x 10^9</td>
<td>24 5.45 x 10^9</td>
<td>29 1.49 x 10^9</td>
</tr>
<tr>
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<td>7 3.65 x 10^9</td>
<td>12 9.68 x 10^9</td>
<td>13 4.48 x 10^9</td>
</tr>
<tr>
<td>Santa Claus</td>
<td>16 9.12 x 10^9</td>
<td>86 2.90 x 10^28</td>
<td>104 8.65 x 10^24</td>
</tr>
<tr>
<td>Dining philosophers</td>
<td>14 4.78 x 10^9</td>
<td>266 8.21 x 10^29</td>
<td>272 5.98 x 10^29</td>
</tr>
<tr>
<td>Cyclic scheduler</td>
<td>18 7.08 x 10^9</td>
<td>100 1.90 x 10^12</td>
<td>106 1.29 x 10^12</td>
</tr>
<tr>
<td>Chiron (original version)</td>
<td>42 6.78 x 10^9</td>
<td>69 4.64 x 10^1</td>
<td>73 5.77 x 10^1</td>
</tr>
<tr>
<td>Chiron (decomposed version 1)</td>
<td>32 5.30 x 10^9</td>
<td>21 1.20 x 10^9</td>
<td>23 1.66 x 10^9</td>
</tr>
<tr>
<td>Chiron (decomposed version 2)</td>
<td>5 3.39 x 10^9</td>
<td>19 2.31 x 10^9</td>
<td>19 2.31 x 10^9</td>
</tr>
</tbody>
</table>

Table 1: The largest size (denoted as “LS”) each algorithm can handle and the number of reachable states at the largest size.

![Runtime Ratio Between ZDD- and BDD-based Algorithms in LTSA Systems](image1)

![Runtime Ratio Between ZDD- and BDD-based Algorithms in FLAVERS Systems](image2)

**Figure 6: Runtime ratios between ZDD- and BDD-based algorithms.**

9. CONCLUSION

FSV techniques can check whether a property holds for a given system. Once the system model and the property are specified, this check is done automatically by exhaustively exploring all possible states of the system. FSV techniques are especially useful for concurrent systems, where nondeterministic behavior usually makes testing problematic. FSV methods, however, are limited by the state-explosion problem. Much research has been done to ameliorate this problem but few comparative studies have been done using the same approach with different symbolic algorithms.

In this paper, we investigate the use of two symbolic data structures, BDDs and ZDDs, in two FSV tools, LTSA and FLAVERS. To do this, we developed an efficient algorithm to compute transition functions with ZDDs. This algorithm treats missing Boolean variables in a ZDD representation in different ways depending on whether the variable is from the domain set of the Boolean function being represented or not. In our experiments, both symbolic algorithms perform much better than the native, non-symbolic algorithms used by LTSA and FLAVERS, in terms of both runtime and the size of the system that the algorithm can handle under the given resource limits. Moreover, the ZDD-based algorithm is almost always better than the BDD-based one. Our results suggest that the ZDD-based approaches should be more widely investigated.

We intend to explore several additional directions. There are several additional variations of the BDD and ZDD based algorithms that appear promising. One variation, for example, is to change the algorithm so that each composite state visited during the check is used in the computation for the transition functions only once. To implement this, besides the function fvisited, another function fnew, which represents the composite states at the current exploration frontier only, is needed. Then only states represented by fnew would be used in the computation for the next frontier. We did a preliminary experiment with this variation and found that sometimes it improves performance, but sometimes it de-
grades performance.

Another variation we want to pursue is to change the encoding scheme for the ZDD representations. The encoding strategy described in this paper uses $\lceil \log_2 n \rceil$ Boolean variables to encode $n$ FSA states. We call this encoding a binary encoding. An alternative, unary encoding would use one variable for each FSA state. A benefit of the unary encoding is that it enables us to use an algorithm for computing transition functions without encoding all transitions in Boolean functions. We also did a preliminary experiment with this approach and found that it also sometimes helps improve performance but sometimes degrades performance. We intend to do more experiments to explore the system features that make one approach more effective than the other.

Besides these variations, we also want to investigate how the BDD and ZDD based algorithms work when the property does not hold (at least for the system model). In that situation, FSV tools can generate a counterexample, a trace through the model showing how the property is violated, that can help the analyst find out what went wrong. For the LTSA search algorithm, it is easy to generate counterexamples once a violation or deadlock is found. For the state propagation algorithm and the symbolic algorithms, however, more work needs to be done on efficiently generating counterexamples.

Finally, as discussed above, we intend to explore the distribution of 1-assignments for a Boolean function. We believe that this will not only help understand why the ZDD representations of the functions we encounter are more compact than the BDD representations, but also may suggest further optimizations.

10. ACKNOWLEDGMENTS

We thank Jeff Magee and Jeff Kramer for kindly providing us with the LTSA source code and example systems.

11. REFERENCES


